P425/2
APPLIED MATHEMATICS
Paper 2
Jul/ Aug 2019
3 hours

MUKONO EXAMINATION COUNCIL

Uganda Advanced Certificate of Education

APPLIED MATHEMATICS

Paper 2

3 hours

INSTRUCTIONS TO CANDIDATES:

Answer **all** *the* **eight** *questions in section***A** *and any* **five** *questions from section***B.**

Any additional question(s) answered will **not** *be marked.*

All*necessary working* **must** *be shown clearly.*

Begin each answer on a sheet of paper.

Squared paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

In numerical work, take acceleration due to gravity, $g = 9.8ms^{-2}$.

SECTION A: (40 MARKS)

Answer **all***the questions in this section.*

1. Kadijat noted the weights, x grams of 30 chocolate buns. Her results are summarized by

   $\sum(x-k) = 315$, $\sum(x-k)^2 = 4022$,

   where$k$ is a constant. The mean weight of the buns is 50.5 grams.

   (i) Find the value of k. ***(03 marks)***

   (ii) Find the standard deviation of x. ***(02 marks)***

2. The acceleration of a particle is $-10\mathbf{j}$. If the particle starts at (0,80) and moving with a velocity of 15**i,**

   a) Find the velocity at time t, t=5s. ***(03 marks)***

   b) Given that at time T, the particle is at (x,0), calculate the values of x and T.

   (02 marks)

3. Use trapezium rule with six strips to estimate; $\int_0^{\pi} \sqrt{(1+\sin x)}\, dx$.

   Truncate your answer correct to **three**significant figures. ***(05 marks)***

4. A car of mass 1000kg working at a constant rate of 160kW is moving with a constant speed of 20m/s up a plane inclined at an angle of 30^0 to the horizontal. Find the magnitude of the resistance to the motion. ***(05 marks)***

5. The resistance of a wire at different temperature is as follows:

Resistance(Ω)	24	42
Temperature (℃)	15	51

   Use linear interpolation or extrapolation to estimate the:

   (i) Temperature corresponding to 35 Ω. ***(03 marks)***

   (ii) Resistance whose value is equal to that of the temperature. ***(02 marks)***

6. A box P contains 3 red and 5 black balls, while another box Q contains 6 reds and 4 black balls. A box is chosen at random and from it a ball is picked and put into another box. A ball is then randomly drawn from the later. Find the probability that;

   (i) Both balls are red. ***(03 marks)***

   (ii) First ball drawn is black. ***(02 marks)***

7. Four forces $\binom{2}{1}N$, $\binom{-1}{3}N$, $\binom{4}{-2}N$ and $\binom{-5}{-2}N$ acts on a particle at (1,1), (2,0), (2,3) and (−1,1) respectively.

   Show that the forces reduce to a couple. ***(05 marks)***

8. The masses of meat cans are normally distributed with a standard deviation of 18g. A random sample of 25 cans had a mean mass of 456g.

   Find the 97.5% confidence interval for the mean mass of all the meat cans.

   (05 marks)

SECTION B: (60 MARKS)

Answers any **five** *questions from this section.*

All *questions carry equal marks.*

9. The numbers of male and female candidates admitted at a certain university in a certain year to offer different courses *A,B,C,D,E,F,G,H,I* and *J* were as follows:

Course	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Male	66	54	60	70	62	46	74	58	80	58
Female	50	38	54	68	60	32	62	46	70	49

a) Calculate the average number of
   i) Males,
   ii) Females,

   Admitted to these courses that year. ***(04 marks)***

b) Represent the given data on a scatter diagram. ***(02 marks)***

c) Determine the rank correlation coefficient for the above data and comment on your result at 0.05 level of significance. ***(06 marks)***

10. a) If the period of a simple harmonic motion is 8s, and the particle oscillates through a distance of 1.2m on each side of the central position. Find;

i) the maximum velocity,
ii) the velocity when the particle is 0.6m from the central position.

(06 marks)

b) A particle is moving with SHM in a straight line and takes 3s to perform a complete oscillation. Its furthest distance from the Centre is 1.2m.

i) Determine its maximum acceleration,

ii) Determine the new amplitude if the particle receives a blow when at its furthest point that drives it with the initial velocity of $0.6\pi\ ms^{-1}$.

(06 marks)

11. a) The numbers $x = 1.5$, $y = -2.85$ and $z = 10.345$ were all rounded off to the given number of decimal places as indicated. Find the range within which the exact value of $\frac{1}{x} - \frac{1}{y} + \frac{y}{xz}$ lies. ***(07 marks)***

b) Two decimal numbers *X* and *Y* were approximated with errors E_1 and E_2 respectively. Show that the maximum possible relative error in the approximation of the product X^2Y is $2\left|\frac{E_1}{X}\right| + \left|\frac{E_2}{Y}\right|$ ***(05 marks)***

12. a) The weights of a group of males are normally distributed with mean 80kg and variance 6.76kg^2. If a random sample of 16 of these men is selected, find the probability that the mean is less than 78.5kg. ***(04 marks)***

b) A football match may be either won or lost by the home team on assumption that no draw is made. The home team is twice as likely to win as to lose the match. If 72 games are played, find the probability that the home team will win;

i) Exactly 50 games,

ii) not more than 40 games.

(08 marks)

13. a) (i) Show that the equation $3^{2x} - 49 = 0$ has a real root between 1 and 2.

(02 marks)

ii) Show that the newton Raphson's formula for approximating the root of the equation is:

$x_{n+1} = \frac{1}{\ln 9}[x_n \ln 9 + 49(3^{-2x_n}) - 1]$ ***(03 marks)***

b) Draw a flow chart that;

i) reads the initial approximation x_0 of the root,

ii) computes and prints the root correct to **three** decimal places.

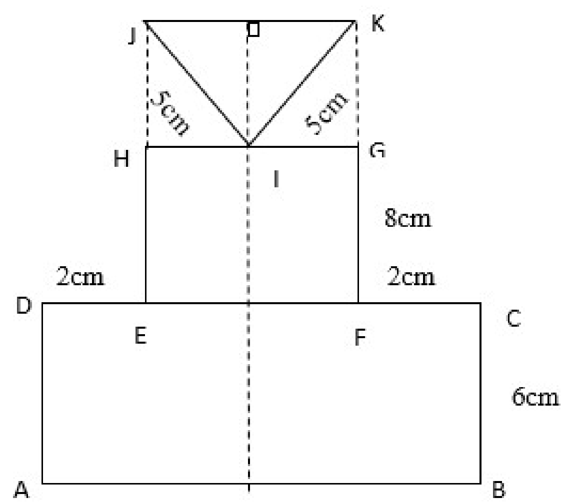
(04 marks)

d) Taking $x_0 = 1.75$, perform a dry run to find the root of the equation. ***(03 marks)***

14. a) Three particles of weights $2W$, $3W$, and $5W$ are located at the points with position vectors $\binom{1}{1}$, $\binom{2}{-3}$, *and* $\binom{4}{1}$ respectively. Find the coordinates of their Centre of gravity.

(04 marks)

b) A composite lamina *ABCDEFGHIJK* is made of a rectangular lamina *ABCD* 12cm by 6cm, a square lamina *EFGH* of side 8cm and triangular lamina *IJK* welded to the square lamina at I as shown below.



i) Find the distance from AB to the position of Centre of gravity of the composite lamina. ***(06 marks)***

ii) If the lamina is suspended from B, find the angle AB makes with the vertical. ***(02 marks)***

15. A game consists of tossing 4 unbiased coins simultaneously. The total score is calculated by giving 3 points for each head and 1 point for each tail. The random variable X represents the total score.

a) Show that the probability of $P(X = 8) = \frac{3}{8}$ ***(04 marks)***

b) Copy and complete the table given below for the symmetrical probability distribution of X. ***(04 marks)***

x	4	6	8	10	12
$P(X = x)$			$\frac{3}{8}$		

c) Calculate the variance of X. ***(04 marks)***

16. Ship A initially at a point with position vector $\binom{4}{2}$ is moving with a speed of 12kmh^{-1} in the direction 30^0 East of North, while ship B initially at a point with position vector $\binom{6}{10}$ is moving with a speed of 4kmh^{-1} due East. Find;

a) the velocity of A relative to B. ***(04 marks)***

b) the shortest distance between the two ships in the subsequent motion and the time for which it occurs. ***(08 marks)***

END